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Fusion procedure and Sklyanin algebra

B Y Hou and Y K Zhou

Institute of Modern Physics, Northwest University, Xian 710069, People's Republic of China

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Abstract. A generalised Sklyanin algebra is studied. The finite-dimensional representations and the centre of this algebra are given with the help of the so-called fusion procedure in this paper.

1. Introduction

The quantum Yang–Baxter equation (QYBE) was first discovered by Yang [1] in the study of a one-dimensional many-body system of spin-$\frac{1}{2}$ fermions interacting by a two-body delta potential and then discovered by Baxter [2] in the study of a zero-field eight-vertex model. A host of investigations revealed that, for finding the solution of integrable models in statistical mechanics and field theory [3–16], the QYBE is a fundamental mathematical relation. By common law we could write the QYBE as

$$R^{12}(u)R^{13}(u+v)R^{23}(v)=R^{23}(v)R^{13}(u+v)R^{12}(u) \tag{1}$$

where u and v are the spectral parameters. The $R^{ij}(u)$ is a linear operator in the tensor product of three linear spaces $V_{123}=V_1\otimes V_2\otimes V_3$ and signifies the operator $R(u)$ on space $V_i\otimes V_j$, acting as the identity on the third space, i.e. $R^{12}(u)=R(u)\otimes\mathbb{1}$. Recently there is also renewed interest in the QYBE because of its connection with some algebraic theories such as the Sklyanin algebra [17, 18], quantum group [19–22] and briad group [23–26].

In the study of the QYBE the so-called fusion procedure was developed to generate new group-invariant solutions of the QYBE-fusion solutions from the known rational solution in [27] and from the Baxter eight-vertex model [2] the so-called Sklyanin algebra was constructed in [17, 18]. Then in [28] from the Belavin $\mathbb{Z}_n\times\mathbb{Z}_n$ symmetric model [29], which is a generalisation of the Baxter eight-vertex model [2], the generalised Sklyanin algebras was raised and the fusion procedure of the elliptic function solution of the QYBE was constructed, and the connection of the finite-dimensional representations of the generalised Sklyanin algebra with the fusion solutions was also pointed out. A detailed derivation of the generalised Sklyanin algebra was given in [30].

The purpose of this paper is to show further work on the generalised Sklyanin algebra. In the next section we review the Belavin $\mathbb{Z}_n\times\mathbb{Z}_n$ symmetric model and the generalised Sklyanin algebra. Then we calculate explicitly the representations of the generalised Sklyanin algebra with the help of the fusion procedure in section 3. In fact, this calculation makes a show of the link of the generalised Sklyanin algebra with the fusion procedure. In section 4 we find the centre of this algebra still using the fusion procedure. In the final section we briefly discuss our results.

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2. The Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model and the generalised Sklyanin algebra

The Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model is the elliptic function solution of the QYBE (1) and it has two equivalent forms [29, 31]:

$$R(u) = \sum S(u)_{ij}^{i'j'} E_{ii'} \otimes E_{jj'} \tag{2a}$$

$$= \exp(-\pi u\sqrt{-1}) \sum W_b(u) I_b \otimes I_b^{-1} \tag{3a}$$

where E_{ij} and I_b are the $n \times n$ matrices, respectively, with the matrix elements

$$(E_{ij})_{i'j'} = \delta_{ii'}\delta_{jj'}$$

and

$$(I_b)_{ij} = \omega^{b_2 j}\sigma_{i,j+b_1}$$

$$\omega = \exp(\sqrt{-1}2\pi/n).$$

The subscript $b = (b_1, b_2)$ belongs in the group $\mathbb{Z}_n \times \mathbb{Z}_n$, where $\mathbb{Z}_n$ is the group of integers $\mathbb{Z}$ modulo n. The summations in (2a) and (3a) are, respectively, over i, j, i', j' and over b_1, b_2 from 0 to $n-1$. The coefficient $S(u)_{ij}^{i'j'}$ is called the Boltzmann weight and has the parametrisation [31]

$$S(u)_{i'j'}^{ij} = \begin{cases} n\exp(-\sqrt{-1}\,\pi u)h(u)\theta\begin{bmatrix}\frac{j-i}{n}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u+w, n\tau) \\ \times\left(\theta\begin{bmatrix}\frac{i'-i}{n}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(w, n\tau)\theta\begin{bmatrix}\frac{j-i'}{n}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u, n\tau)\right)^{-1} & \text{if } i+j=i'+j' \bmod n \\ 0 & \text{otherwise} \end{cases} \tag{2b}$$

$$h(u) = \prod_{i=0}^{n-1}\theta\begin{bmatrix}\frac{i}{n}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u, n\tau)\left(\prod_{i=1}^{n-1}\theta\begin{bmatrix}\frac{i}{n}+\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(0, n\tau)\right)^{-1}$$

which can be obtained from the Belavin parametrisation [29]

$$W_b(u) = \theta\begin{bmatrix}b_1/n+\frac{1}{2}\\ b_2/n+\frac{1}{2}\end{bmatrix}(u+w/n, \tau)\left(\theta\begin{bmatrix}b_1/n+\frac{1}{2}\\ b_2/n+\frac{1}{2}\end{bmatrix}(w/n, \tau)\right)^{-1}. \tag{3b}$$

They are all represented in terms of the Jacobi theta function:

$$\theta\begin{bmatrix}a\\ b\end{bmatrix}(u, \tau) = \sum_{m\in\mathbb{Z}} \exp\{\sqrt{-1}\,\pi[(m+a)^2\tau + 2(m+a)(u+b)]\}.$$

The parameter w is a constant.

The Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model is the generalisation of the Baxter eight-vertex model. The diagonalisation of the transfer matrices for the Baxter eight-vertex model has been given in [4, 7] and for the Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model it has been given in [8, 32]. The fusion procedure for these models has also been studied in [28, 33–35]. Notably, since the so-called Sklyanin algebra was constructed from the Baxter eight-vertex model in [17, 18], the generalised Sklyanin algebra has been obtained recently from the Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model in [28, 30]. In the following we describe briefly the derivation of the generalised Sklyanin algebra [30].

Introducing operators $A_b \in V_3$ and

$$L(u) = \sum_{b \in \mathbb{Z} \times \mathbb{Z}} W_b(u) I_b \otimes A_b \tag{4}$$

and using the solution (3) of the QYBE (1), we construct the following relation:

$$R^{12}(u) L^{13}(u+v) L^{23}(v) = L^{23}(v) L^{13}(u+v) R^{12}(u). \tag{5}$$

Using the relation

$$\operatorname{Tr} I_a I_b^{-1} = n\delta_{ab}$$

from (5) we have

$$\sum_{c \in \mathbb{Z} \times \mathbb{Z}} F_{abc}(u, v) \omega^{(a_1 - c_1)(c_2 - b_2)} A_{a+b-c} A_c = 0 \tag{6}$$

where the coefficient

$$F_{abc}(u, v) = W_{c-b}(u) W_{a+b-c}(u+v) W_c(v) - W_{a-c}(u) W_c(u+v) W_{a+b-c}(v)$$

can be represented as

$$F_{abc}(u, v) = C_{abc} f_{ab}(u, v)$$

$$f_{ab}(u, v) = \theta\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(u, \tau)\, \theta\begin{bmatrix} a_1/n + \frac{1}{2} \\ a_2/n + \frac{1}{2} \end{bmatrix}(u + v + 2w/n, \tau)\, \theta\begin{bmatrix} b_1/n + \frac{1}{2} \\ b_2/n + \frac{1}{2} \end{bmatrix}(v, \tau)$$

using an elementary complex analysis argument [36]. The constant C_{abc} is independent of the spectral parameters u and v and is only dependent on the parameters w and τ. It can be calculated by considering the ratio of F_{abc} to $f_{ab}(u, v)$. Here we do not go further into this and the detailed calculation has been given in [30]. Finally, from (6) we have

$$\sum_{c \in \mathbb{Z} \times \mathbb{Z}} C_{abc} A_{a+b-c} A_c = 0 \qquad \text{for any } a, b, c \in \mathbb{Z}_n \times \mathbb{Z}_n. \tag{7}$$

Equations (7) are independent of the spectral parameters u and v and define an algebra. This algebra is just the Sklyanin algebra [17, 18] for the special case of $n = 2$. Hence equations (7) define the generalisation of the Sklyanin algebra—the generalised Sklyanin algebra.

3. Fusion procedure and the representations of the generalised Sklyanin algebra

The fusion procedure of the Belavin $\mathbb{Z}_n \times \mathbb{Z}_n$ symmetric model was systematically studied in [28, 33-35, 37]. In this section we would like to find some finite-dimensional representations of the generalised Sklyanin algebra explicitly using the fusion procedure.

Let N be a positive integer, $\bar{V}_i = V_j = C^n$ and $V^{\otimes N} = V_1 \otimes \ldots \otimes V_N$. Take $R^{ij}(u)$, $R^{\bar{i}j}(u)$ and $R^{\bar{i}\bar{j}}(u)$ as, respectively, the operators $R(u)$ acting on $V_i \otimes V_j$, $\bar{V}_i \otimes V_j$ and $\bar{V}_i \otimes \bar{V}_j$. Then we define the following operator acting on $\bar{V} \otimes V^{\otimes N}$ using (2):

$$R_N(u) = R^{\bar{1}1}(u) R^{\bar{1}2}(u+w) \ldots R^{\bar{1}N}(u + (N-1)w) \tag{8a}$$

and

$$R_{\sigma N}(u) = P_N^{\sigma} R_N(u) P_N^{\sigma} \tag{8b}$$

where P_N denotes the projector on the space of symmetric tensors and antisymmetric tensors in $V^{\otimes N}$, respectively, for $\sigma=+$ and $\sigma=-$. Here we take $(P_N^\sigma)^2=P_N^\sigma$.

It has been shown in [35] that $R_{\sigma N}(u)$ satisfies the following YBE:

$$R^{12}(u)R^{13}_{\sigma N}(u+v)R^{23}_{\sigma N}(v)=R^{23}_{\sigma N}(v)R^{13}_{\sigma N}(u+v)R^{12}(u). \tag{9}$$

This relation is similar to (5) in form. This shows that $R_{\sigma N}(u)$ should be the finite-dimensional representation of the operator $L(u)$ in space $V\otimes V^{\otimes N}$. Therefore we can obtain the finite-dimensional representation of the algebraic elements A_b from the fusion solution $R_{\sigma N}(u)$. We may write

$$R_{\sigma N}(u)=\sum_{b\in\mathbb{Z}\times\mathbb{Z}} I_b F_b^{\sigma N}(u), \tag{10}$$

where the operators $F_b^{\sigma N}(u)\in V^{\otimes N}$ are dependent on the spectral parameter u. Thus far our task is to find it explicitly.

Using (2) for any $s=(s_1,s_2)\in\mathbb{Z}\times\mathbb{Z}$ we have

$$S(u+s_1\tau+s_2)^{ij}_{i'j'}=\exp\left\{-\sqrt{-1}\,\pi\left[(s_1^2\tau+2s_1\left(u+\frac{w}{n}+\frac{\tau+1}{2}\right)\right]\right\}\omega^{s_2(i'-i)}S(u)^{ij+s_1}_{i'j'+s_1}.$$

Inserting this into (8), and with the help of the representation (10), we obtain

$$F_b^{\sigma N}(u+\tau)=\exp\left\{-N\pi\sqrt{-1}\left[\tau+2\left(u+\frac{w}{n}+\frac{(N-1)w+\tau+1}{2}\right)\right]\right\}\omega^{-b_2}F_b^{\sigma N}(u)$$

$$F_b^{\sigma N}(u+1)=\omega^{b_1}F_b^{\sigma N}(u).$$

Since $R(u)$ and $F_b^{\sigma N}(u)$ are entire, these transformation properties show that the operator $F_b^{\sigma N}$ has N zeros within the parallelogram $\Lambda_\tau=1+\tau$ of the complex plan, and if u_i^σ $(0\leqslant i\leqslant N-1)$ are the zeros, they satisfy

$$u_0^\sigma+\ldots+u_{N-1}^\sigma=-Nw\left(\frac{1}{n}+\frac{N-1}{2}\right)-\frac{1}{n}(b_2+b_1\tau).$$

These can easily be obtained from an argument of the elementary complex analysis [36].

Since equation (2) gives [31, 35]

$$R(-w)=\text{something}\times P_2^-$$

and

$$R(w)=P_2^+\times\text{something}$$

it is obvious from (8) that we can locate the first $N-1$ zeros of the $F_b^{\sigma N}(u)$ as follows:

$$u_i^+=-iw \qquad i=1,2,\ldots,N-1$$

and

$$u_i^-=-iw \qquad i=0,1,\ldots,N-2.$$

From the above sum of zeros we conclude the remaining zero must be

$$u_0^+=-\frac{N}{n}w-\frac{1}{n}(b_2+b_1\tau) \qquad \text{if } \sigma=+$$

and

$$u_{N-1}^-=-\left(\frac{N}{n}+N-1\right)w-\frac{1}{n}(b_2+b_1\tau) \qquad \text{if } \sigma=-$$

Define

$$H_{\sigma N}(u)=\begin{cases}\exp(-\sqrt{-1}\,\pi N u)\prod_{i=1}^{N-1}\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u+iw,\tau) & \text{if } \sigma=+ \quad (11a)\\ \exp(-\sqrt{-1}\,\pi N u)\prod_{i=0}^{N-2}\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(u+iw,\tau) & \text{if } \sigma=- \quad (11b)\end{cases}$$

$$W_b^{\sigma N}(u)=\begin{cases}W_b[u+(N-1)w/n] & \text{if } \sigma=+ \quad (11c)\\ W_b[u+(N-1)(w/n+w)] & \text{if } \sigma=- \quad (11d)\end{cases}$$

then the product $H_{\sigma N}(u)W_b^{\sigma N}(u)$ has the same zeros (in u) as $F_b^{\sigma N}(u)$ and the same transformation properties as $F_b^{\sigma N}(u)$. Thus

$$F_b^{\sigma N}(u)=H_{\sigma N}(u)W_b^{\sigma N}(u)A_b^{\sigma N} \tag{12}$$

where we have introduced the operators $A_b^{\sigma N}\in V^{\otimes N}$. They are independent of the spectral parameter u. In fact, these $A_b^{\sigma N}$, $b\in\mathbb{Z}_n\times\mathbb{Z}_n$, $N=$ any positive integers, are just the finite-dimensional representations of the generalised Sklyanin algebraic elements A_b ($b\in\mathbb{Z}_n\times\mathbb{Z}_n$) in (7). We use the notation rep(σ, N) to denote the representation with $A_b^{\sigma N}$. This can be obviously seen from inserting (12) in (10) and from (9) we have the same commutation relation for $A_b^{\sigma N}$ as (7) for A_b. To determine $A_b^{\sigma N}$ for any $N>1$ we consider $R_{\sigma N}(u)$. From (8) we have

$$R_{\sigma N}(u)=P_N^{\sigma}R^{\bar{1}1}(u)R_{\sigma(N-1)}^{\bar{1},2\ldots M}(u+w)P_N^{\sigma} \tag{13a}$$

or

$$R_{\sigma N}(u)=P_N^{\sigma}R_{\sigma(N-1)}^{\bar{1},1\ldots N-1}(u)R^{\bar{1}N}(u+(N-1)w)P_N^{\sigma}. \tag{13b}$$

Inserting (3) and (10)-(12) in (13a), we can obtain

$$A_b^{+N}=u_b^{+}(N-1)\sum W_c(w+(N-2)w/n)\omega^{(b_2-c_2)c_1}P_N^{+}I_{b-c}^{-1}\otimes A_c^{+(N-1)}P_N^{+} \tag{14a}$$

$$u_b^{+}(N-1)=\left[\exp[\sqrt{-1}\,\pi(N-1)w]\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(w,\tau)W_b\left(\frac{w}{n}(N-1)\right)\right]^{-1}. \tag{14b}$$

Inserting the same equations in (13b), we have

$$A_b^{-N}=u_b^{-}(N-1)\sum W_c(-w+(N-2)w/n)\omega^{c_2(b_1-c_1)}P_N^{-}A_c^{-(N-1)}\otimes I_{b-c}^{-1}P_N^{-} \tag{14c}$$

$$u_b^{-}(N-1)=\left[\exp[\sqrt{-1}\,\pi(N-1)w]\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}(-w,\tau)W_b\left((N-1)\frac{w}{n}\right)\right]^{-1} \tag{14d}$$

where the summations are $c=(c_1,c_2)$ over $\mathbb{Z}_n\times\mathbb{Z}_n$. The factor $W_b(u)$ is given in (3b), and

$$A_b^{\sigma 1}=I_b^{-1} \qquad \text{for } \sigma=\pm. \tag{14e}$$

Equations (14) give a set of recurrence relations for determining $A_b^{\sigma N}$. Thus we have the final results for any $N\geqslant 1$

$$A_{b^0}^{+N}=\sum_{b^1}\cdots\sum_{b^{N-1}}\left(\prod_{i=1}^{N-1}\{u_{b^{i-1}}^{+}(N-i)W_{b^i}[w+(N-i-1)w/n]\omega^{b_1^i(b_2^{i-1}-b_2^i)}\}\right.$$
$$\left.\times P_N^{+}I_{b^0-b^1}^{-1}\otimes I_{b^1-b^2}^{-1}\otimes\ldots\otimes I_{b^{N-1}}^{-1}P_N^{+}\right) \tag{15a}$$

and

$$A_{b^0}^{-N} = \sum_{b^1} \ldots \sum_{b^{N-1}} \left(\prod_{i=1}^{N-1} \{ u_{b^{i-1}}(N-i) W_{b^i}[-w(N-i-1)w/n] \omega^{b_2^i(b_1^{i-1}-b_1^i)} \} \right.$$

$$\left. \times P_N^- I_{b^{N-1}}^{-1} \otimes \ldots \otimes I_{b^1-b^2}^{-1} \otimes I_{b^0-b^1}^{-1} P_N^- \right). \qquad (15b)$$

They give the representation rep(σ, N) of the generalised Sklyanin algebra. Since I_b^{-1} are the $n \times n$ matrices, A_b^{-N} becomes zero if $N > n$ and a 1×1 matrix (a non-matrix function) if $N = n$ in equation (15b). In addition, the recurrence relation for $A_b^{\sigma N}$ defines the following mapping:

$$\Delta_N^\sigma \colon \text{rep}(\sigma, N) \to \text{rep}(\sigma, N+1).$$

4. Fusion procedure and the algebraic centre

In this section we find the centre of the generalised Sklyanin algebra by using the fusion procedure.

Define an operator acting on $V^{\otimes N}$ for any $N \geqslant 1$:

$$D_{\sigma N}(u) = P_n^- R_{\sigma N}^{\bar{n}1}(u + (1-n)w) \ldots R_{\sigma N}^{\bar{2}1}(u - w) R_{\sigma N}^{\bar{1}1}(u) P_n^- \qquad (16)$$

where $R_{\sigma N}^{\bar{i}1}(u)$ acting on $\bar{V}_i \otimes V_1 = V \otimes V^{\otimes N}$ is given in (8). P_n^- is a projector on the space of antisymmetric tensors $\bar{V}^{\otimes N}$. It has been shown in [35] that we have the following YBE:

$$R_{-n}^{12}(u) R_{\sigma N}^{13}(u+v) D_{\sigma N}^{23}(v) = D_{\sigma N}^{23}(v) R_{\sigma N}^{13}(u+v) R_{-n}^{12}(u). \qquad (17)$$

Now we will show that the operator $D_{\sigma N}(v)$ is the centre element of the representation rep(σ, N) of the generalised Sklyanin algebra. To show this we must study the operator $R_{-n}^{12}(u)$.

Using (2) for $s = (s_1, s_2) \in \mathbb{Z} \times \mathbb{Z}$ we have

$$R(u + s_1\tau + s_2) = P(u) \mathbb{1} \otimes I_s^{-1} R(u) \mathbb{1} \otimes I_s$$
$$= P(u) I_s \otimes \mathbb{1} R(u) I_s^{-1} \otimes \mathbb{1}$$

where $\mathbb{1}$ is a unit matrix. The coefficient $P(u)$ is a factor depending on the spectral parameter u and is not important in our further discussion. From this transformation and (8) we have

$$R_{-n}(u + s_1\tau + s_2) = \prod_{i=0}^{N-1} P(u + iw) \mathbb{1} \otimes U_s^{-1} R_{-n}(u) \mathbb{1} \otimes U_s \qquad (18a)$$

$$= \prod_{i=0}^{N-1} P(u + iw) I_s \otimes \mathbb{1} R_{-n}(u) I_s^{-1} \otimes \mathbb{1} \qquad (18b)$$

$$U_s = I_s \otimes \ldots \otimes I_s \qquad (nI_s).$$

Since the representation A_b^{-n} is the 1×1 matrix, the operator $\mathbb{1} \otimes U_s$ commutes with $R_{-n}(u)$. Therefore the transformation (18) gives

$$R_{-n}(u) = I_s \otimes \mathbb{1} R_{-n}(u) I_s^{-1} \otimes \mathbb{1}. \qquad (19)$$

Moreover, because we have $i + j = i' + j' \bmod n$ for $S(u)_{ij}^{i'j'}$ and A_b^{-n} is a 1×1 matrix, $R_{-n}(u)$ is a $n \times n$ diagonal matrix. Combining this and (19) we know that $R_{-n}(u)$ must be product of something and a unit matrix. Thus (17) gives

$$[R_{\sigma N}^{13}(u+v), D_{\sigma N}^{23}(v)] = 0.$$

Using (10), (12) and

$$\mathrm{Tr}\, I_a I_b^{-1} = n\delta_{ab}$$

we have

$$[A_b^{\sigma N}, D_{\sigma N}(v)] = 0 \qquad \text{for any } b \in \mathbb{Z}_n + \mathbb{Z}_n. \tag{20}$$

This commutation relation shows that the operator function $D_{\sigma N}(u)$ for any complex variable u is the centre element of the representation rep(σ, N) of the generalised Sklyanin algebra.

5. A brief discussion

In the previous section we have constructed the finite-dimensional representations and the centre of the generalised Sklyanin algebra. In fact, we have also found the fusion solution $R_{\sigma N}(u)$, which can be obtained by equations (10)-(12) and (14). But there are still some related interesting questions to be studied.

(i) Is the representation (14), taking the special values of w as a restricted sos model [33], reducible?

(ii) How many centre elements are there in the generalised Sklyanin algebra?

(iii) Can we construct the representations with the other symmetries for the generalised Sklyanin algebra?

These, however, have not been studied in this paper, but will be the subject of our future study.

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